MODULE - 4

MAGNETISM & ELECTROMAGNETIC THEORY

A stationary charge produces an electrostatic field ($E \neq 0, B = 0$). An electric charge in steady motion give rise to both electric and magnetic fields ($E \neq 0, B \neq 0$). An accelerating electric charge give rise to a radiation field or EM waves ($B \neq 0, E \neq 0$).

* Magnetic flux ϕ : A magnetic field is characterized by its field lines. If field is strong, these field lines are more closely spaced. Therefore the no. of magnetic field lines through a surface is a measure of the strength of the magnetic field there. ie, total no. of magnetic lines of force produced is called magnetic flux, ϕ.

* Magnetic flux density (**B**) : It is defined as the flux passing through unit area $B = \phi / A$.
It's unit is weber per m^2, (Wb/m^2), or Tesla(T).

Also, **B** is the maximum force experienced by 1 coulomb charge moving with velocity 1 m/s.

$$F = q(v \times B) \rightarrow \text{Lorentz force.}$$

The magnetic flux density is said to be one tesla if the maximum force experienced by 1 C charge moving with velocity 1 m/s is 1 Newton.

* Magnetic Field Intensity or Magnetising Field (H):

Magnetic field strength stated for free space is called magnetising field or magnetic field intensity (H). H represents the magnetic field produced by external current and do not incorporate the response of the medium to this field. It's unit is Ampere/metre (A/m).

B and H are related by the eqn,

$$B = \mu_m H$$

where μ_m is the permeability of the medium.
[permeability : Ability of a material to conduct magnetic lines of force through it.]

* Magnetization (M) :

$$M = \frac{\text{net dipole moment}}{\text{Volume}}$$

$$= \frac{I \times Area}{Volume} = \frac{Ampere \times m^2}{m^3}$$

$$= A/m$$

* Magnetic susceptibility (χ) :

It is the ratio of Magnetization to the magnetising field H.

$$\chi = \frac{M}{H} = \frac{A/m}{A/m} \Rightarrow \text{dimension less.}$$

* Magnetic permeability (μ) : The ability of the material to support the formation of magnetic

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field within it. it's unit is henry/metre (H/m)

$\mu = B/H$

permeability of free space (vaccum) is μ_0

$\mu_0 = 4\pi \times 10^{-7}$ H/m

We know that, total flux density B is,

$B = \mu_0 H + \mu_0 M$

$= \mu_0 (H + M)$

$B = \mu_0 (H + \chi_m H)$

$B = \mu_0 (1 + \chi_m) H \longrightarrow (1)$

$\mu_0 H \rightarrow$ due to external field

$\mu_0 M \rightarrow$ due to magnetization of material.

$M = \chi_m H$

χ_m = susceptibility of medium

Generally $B = \mu H \longrightarrow (2)$

comparing (1) & (2) $\Rightarrow$ $\mu = \mu_0 (1 + \chi_m)$

$\frac{\mu}{\mu_0} = (1 + \chi_m)$

$\therefore \mu_r = (1 + \chi_m)$

where μ_r is the relative permeability.

$\mu_r = \frac{\mu}{\mu_0} = \frac{\text{Permeability of medium}}{\text{Permeability of free space}}$

μ_r is dimension less.

* Magnetic Materials

Magnetic effect of matter is caused by spin and orbital motion of electrons. The spin and orbital dipole moments of electron add up vectorially and the resultant magnetic moment of each atom again combine with one another producing a net magnetic dipole moment in the material. The magnetic properties of the material is due to the interaction of this net magnetic dipole moment with the external field. When the net dipole moment (spin + orbital) is zero or cancel each other then the substance is nonmagnetic.

classification of magnetic materials,

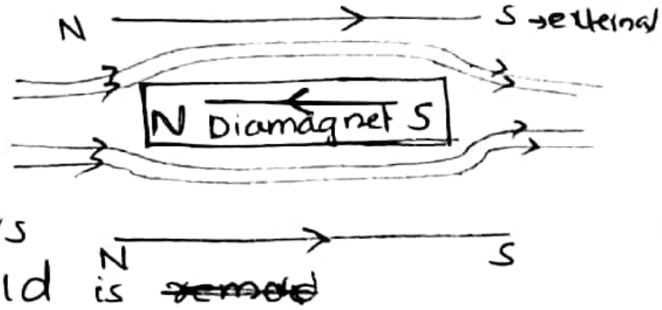
(i) Diamagnetic
(ii) Paramagnetic
(iii) Ferromagnetic

* Diamagnetic substance

- In diamagnetic substance, the resultant magnetic moment due to all atomic current loop is zero in the absence of an external magnetic field.

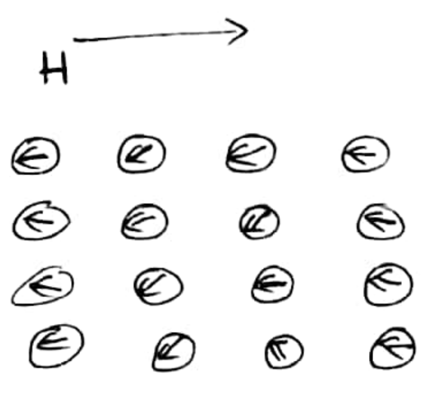
- But in the presence of an external magnetic field, weak magnetic dipole moments are produced in the atoms.

- So a net magnetic dipole moment is produced and which is opposite to the direction of external field.



- The net magnetic dipole moment disappears when the external field is ~~remod~~ removed.

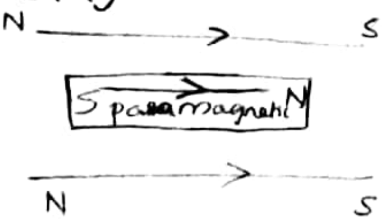
- So a diamagnetic material placed near to magnet is repelled by it.
- Since magnetization produced inside the material is opposite to the external field, then χ is negative.
- χ is negative then μ_r is slightly less than one.
- Induced magnetic dipole moment and χ of a diamagnetic substance is Independent of temp.
- Diamagnetic substance expells magnetic field lines from its interior.
- Superconductors exhibit perfect diamagnetism whose χ value is -1, which is very large. Because of this it finds its application in magnetic levitation.
- When a diamagnetic rod is freely suspended in an external magnetic field, it stays $\perp$cular to the magnetic field lines.



- Eg: Water, Bismuth, gold, silver. etc
- If the field is non-uniform, it tend to move towards the region of weaker field.

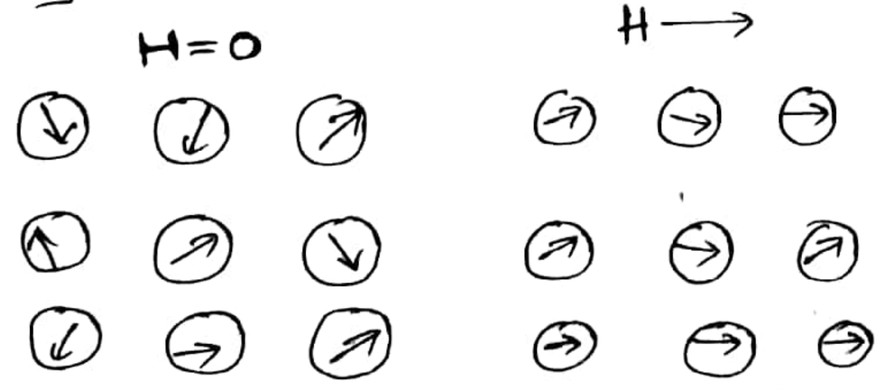
* Paramagnetic substance.

- In materials exhibiting paramagnetism, the spin and orbital magnetic moments in an atom do not cancel each other; instead they add vectorially.
- There will be a net dipole moment in each atom.
- In the absence of an external magnetic field, these atomic magnetic moments are oriented in random manner so that the net magnetic moment of the material is zero.
- But when an external magnetic field is applied, these magnetic moments tend to align with the external field (along the direction of external field).



* Substance which develop a resultant magnetic moment in the direction of an applied external magnetic field can be termed as paramagnetic

* It susceptibility is small and positive and relative permeability is slightly greater than unity.



- When a paramagnetic rod is freely suspended it aligns itself along with field lines.

- Magnetization (M) of a paramagnetic material is directly proportional to the external magnetic field (H) and inversely proportional to the temperature

- Eg: chromium, platinum, magnesium, oxygen, etc

* Ferromagnetic substance

- In ferromagnetic materials, due to a quantum mechanical effect called exchange coupling the spins, of the atoms interact with one another and as a result, there will be alignment of the magnetic dipole moments. This effects leads to small domain of uniform magnetization.

- All atomic dipole moments within a domain are lined up. Hence there is a net dipole moment for every domain. But the orientation of moment varies from domain to domain.

- In unmagnetized ferromagnetic materials, these moments are distributed randomly then bulk magnetic moment is zero.

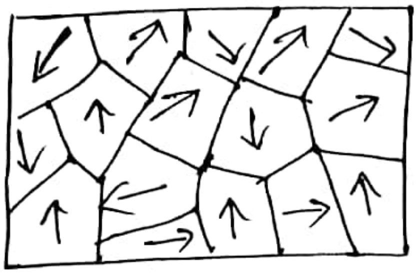
- On the application of an external magnetic field, the domain orient in the direction of the field.

- They have very high magnetic susceptibility and permeability.

- M and H are not linearly related.

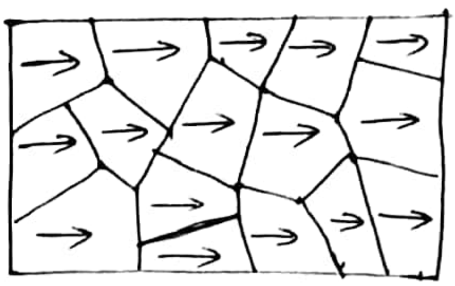
- These materials can be easily magnetised to a great extent. Magnetic Field in a region can be enhanced by a Ferromagnetic material.
- Ferromagnetic materials show hysteresis.
- Eg: Iron, nickel, cobalt, ...

H = 0

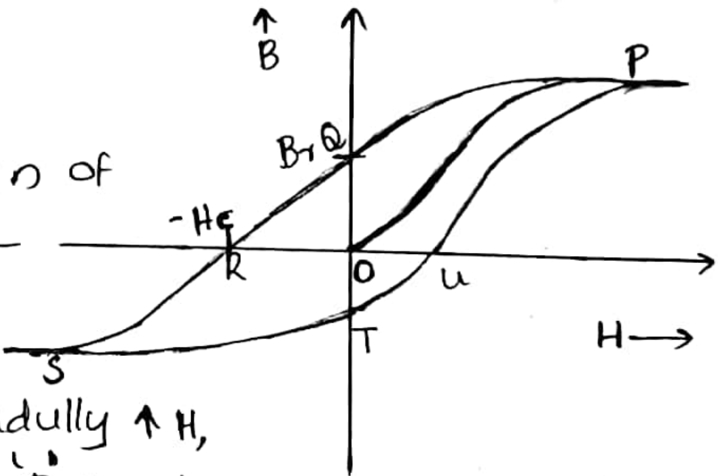


Ferromagnetic material - in each domain there is a net magnetization even in the absence of an external field.

H →



In the presence of an external field, the magnetic moments of domains align in the field direction



* Hysteresis :

consider a specimen of unmagnetised ferro-magnetic material.

Initially B and H are zero. By gradully ↑ H, B also increases. 'P' is the saturation point ie, further increase in H does not produce an increase in B.

- On decreasing H, B does not retrace the original path OP, But follows PQ. When H = 0, B ≠ 0. ie, B_r is called retentivity of the substance.

* On reversing and ↑ H, B=0 at $H = H_c$.
  Where H_c ⇒ coercive Force [It is the minimum magnetising field necessary to reduce B to zero].
* On further ↑ in H we get saturation point 'S' in opposite direction.
* then ↓ H we get a point 'T' ie retentivity in opposite direction.
* On ↑ H, B varies and get a loop.
* Here B is lagging behind H. This phenomenon is hysteresis.

Applications of Ferromagnetic materials,

(i) For making permanent magnets (materials with high H_c)

(ii) Ferromagnetic materials are used for strengthening the magnetic Field in a region (eg: cores in transformers, cores in electromagnets)

Some important Vector identities

1. $\nabla\cdot(\nabla\phi) = \nabla^2\phi$ ie div(grad ϕ) = del^2 ϕ
2. $\nabla\times(\nabla\phi) = 0$ ie curl(grad ϕ) = 0
3. $\nabla\cdot(\nabla\times\vec{A}) = 0$ ie, div(curl A) = 0
4. curl (curl A) = grad(div A) − del^2 A

   ie, $\nabla\times(\nabla\times A) = \nabla(\nabla\cdot A) - \nabla^2 A$

Diamagnetic	Paramagnetic	Ferromagnetic
The individual atoms or molecules have no net magnetic dipole moment in the absence of an external field.	The individual atoms or molecules have a net magnetic dipole moment in the absence of an external field.	The individual atoms or molecules have a net magnetic dipole moment and these atomic magnets organises into domains in the absence of an external field.
They are weakly repelled by a magnet	They are weakly attracted by a magnet	They are strongly attracted by a magnet.
They try to expel the magnetic field lines when placed in an external field and the resultant field within the material is reduced.	They try to concentrate the magnetic field lines within them when placed in an external field and the resultant field within the material is enhanced	The magnetic field lines are highly concentrated within them when placed in an external field and the resultant field is strongly enhanced.
They tend to move from a region of strong field to a region of weak field when placed in non uniform field	They tend to move from a region of weak field to a region of strong field when placed in a non uniform field.	They tend to move from a region of weak field to a region of strong field when placed in non uniform field.
Suscpetibility is negative	Susceptibility is small and positive	Susceptibility is large positive value.
Suscpetibilityis independent of temperature and the substances does not obey Curie law.	Susceptibility varies inversly with temperature and the substance obeys Curie law.	Susceptibility varies inversely with temperature and above Curie temperature the ferromagnetic substance becomes paramagnetic Above Curie temperature,the substance obeys Curie-Weiss law.
Relative permeability is less than unity	Relative permeability is slightly greater than unity.	Relative permeability is greater than unity.
Do not exhibit the phenomenon of Hysteresis	Do not exhibit the phenomenon of Hysteresis	Exhibit the phenomenon of Hysteresis
Examples:Gold, Copper, Antimony, Bismuth , Lead, Quartz, Air, Hydrogen, Water, Alcohol, Sodium Chloride etc.	Examples; Platinum , Aluminium , Lithinum, Magnesium, Chromium , Copper Chloride etc.	Examples : Iron , Nickel, Cobalt, Steel, Alnico etc.

Gauss Law for Magnetic Flux Density.

We know that magnetic flux $\phi = BA$
where B = flux density and A = Area.

Flux linked with a small surface $d\vec{s}$

is, $d\phi_B = B_{perpendicular}\, ds$

$= B\cos\theta\, ds$ ✗

$d\phi_B = \vec{B}\cdot d\vec{s}$

$B_{perpendicular}$ = $\perp$cular component of B

ds = elementary area.

Total flux through the surface is, $\phi_B = \int \vec{B}\cdot d\vec{s} = \iint \vec{B}\cdot d\vec{s}$

We know that the lines of B are continuous. There is no isolated monopoles. The magnetic flux that leaves the volume (out flux) is equal to the flux that enters (influx) the volume.

According to Gauss law,

Flux linked with a closed surface in magnetic field is zero.

$$\boxed{\oint \vec{B}\cdot d\vec{s} = 0} \rightarrow \text{Integral form of Gauss law.}$$

Using Gauss-divergence theorem,

$$\oint_s \vec{B}\cdot d\vec{s} = \int_v (\nabla\cdot B)\, dv = 0$$

$$\therefore \boxed{\nabla\cdot B = 0} \quad \text{Gauss law in differential form.}$$

$\therefore$ B is solenoidal.

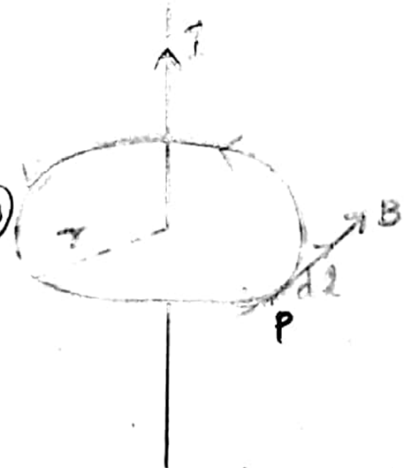
Ampere's Circuital Law

Consider a long straight conductor carrying current I. The magnetic field lines encircle the conductor. The direction of the magnetic field **B** is given by Right hand rule.

Consider a circular (closed) path around the conductor with radius r. Since r is constant throughout the path, B is also constant around the circular path (Amperian loop).

The magnetic field B at a point r from the conductor,

$$B = \frac{\mu_0 I}{2\pi r}$$ (Biot-Savart law)



Ampere's circuital law:

The line integral of B around a closed path is equal to μ_0 times the total current enclosed by the path.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

It is important to note that Ampere's circuital law is valid only if the magnetic field is steady.

Consider an elementary length dl on the Amperian loop. At point P, $\vec{B}$ and

$\vec{dl}$ are tangential and parallel.

$$\oint \vec{B}\cdot\vec{dl} = \oint B\,dl\cos\theta = \oint B\,dl \quad \Big|\ \because \cos\theta = 1$$

$$= B\oint dl$$

$$= \frac{\mu_0 I}{2\pi r}\,[2\pi r]$$

$$\therefore \oint \vec{B}\cdot\vec{dl} = \mu_0 I$$

Faraday's law of Electromagnetic Induction

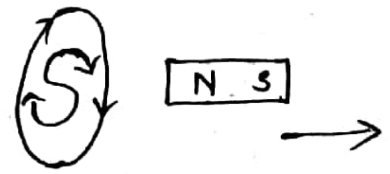
Law I : Change in magnetic flux linked to a coil induce an emf across a coil.

Law II : The emf induced across the coil is equal to rate of change of flux in the coil.

$$e = -\frac{d\phi}{dt}$$

| -ve sign shows that emf induced always oppose the change in flux.

Lenz's law :- To determine the direction of induced induced emf in a loop. The direction of induced current will be in such a way that the magnetic field produced by it opposes the ~~flux~~ change of flux that actually induces the current.



ϕ increased induced to make flux is opposite direction.

ϕ decreased induced to make flux is same direction.

ELECTROMAGNETIC THEORY

James clerk Maxwell combine electric and magnetic phenomena for explaining the properties of electromagnetic waves. In his attempt to unify the laws of electricity and magnetism Maxwell formulated four eqns, thats are known after his name.

Scalar function and scalar field

If every point (x, y, z) in a region of space R corresponds to a scalar quantity f, then the scalar quantity is written as $f(x, y, z)$ and is known as scalar point function. The region of space over which the function is defined is known as scalar field.

eg: The temperature of the atmosphere

The electric potential around a charged body.

Vector function and vector field

If every point in a region of space R corresponds to a vector quantity f, then the vector quantity is written as $f(x, y, z)$

and is known as vector function. The region of space over which the function is defined is known as vector field.

eg: The wind velocity in a room/atmosphere
The force on a charged body placed in an electric field.

Operator ∇

∇ - del is a differential operator capable of differentiating both vector and scalar functions.

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

when ∇ operator act on a scalar function T, then ∇T (the gradient)

when ∇ operator act on a vector function $\vec{V}$, then $\nabla \cdot \vec{V}$ (the divergence)

when ∇ operator act on vector function $\vec{V}$, then $\nabla \times \vec{V}$ (the curl)

Gradient and its physical significance

If $\phi(x, y, z)$ is a scalar function of variables x, y, z the product of 'del' with ϕ is known as 'gradient of the scalar function' ϕ and is abbreviated as grad ϕ.

$$\text{grad } \phi = \nabla\phi = \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}$$

$\nabla\phi$ is vector

The gradient of a scalar function gives the unique direction in which the scalar function changes most rapidly, i.e the maximum space rate of change of that function.

If T is the temperature in a room, it depends on three variables x, y, z ie $T(x, y, z)$.

Then, $$dT = \frac{\partial T}{\partial x}dx + \frac{\partial T}{\partial y}dy + \frac{\partial T}{\partial z}dz$$

Also $$dT = \left(\frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}\right)\cdot\left(\hat{x}dx + \hat{y}dy + \hat{z}dz\right)$$

$$dT = (\nabla T)\cdot(d\vec{l})$$

$$dT = |\nabla T|\,|d\vec{l}|\cos\theta$$

θ is the angle b/w ∇T and $d\vec{l}$. If we fix the magnitude $|d\vec{l}|$ and search around in various directions (ie vary θ), then maximum change in T evidently occur when $\theta = 0$ (then $\cos\theta = 1$). That is, for a fixed distance $|d\vec{l}|$, dT is greatest when we move in the same direction as ∇T. ie, The gradient ∇T points in the direction of ~~max~~ maximum increase of the function T. The magnitude $|\nabla T|$ gives the slope (rate of increase) along this maximal direction.

examples; i) Temperature gradient in heat conduction
ii) Velocity gradient in viscous flow.
iii) In electrostatics' $E = -\nabla V$. This means that the intensity of electric field is directed along the maximum rate of decrease of potential with distance.

Divergence and its physical significance

The dot product of vector differential operator, 'del' with a vector function is known as 'divergence of the vector function'. ie 'div' of the function.

If $\vec{A}(x,y,z)$ be a vector function,

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

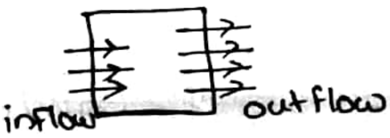
$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$\vec{\nabla} \cdot \vec{A}$ is a scalar quantity.

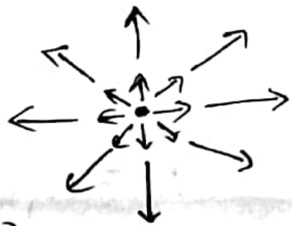
$\nabla \cdot \vec{A} = 0$ means $\vec{A}$ is solenoidal.

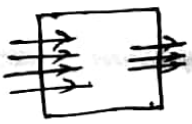
Divergence of a vector function gives the net outflow (outflow minus inflow) per unit volume at a point. Also $\nabla \cdot \vec{A}$ measure how much the vector $\vec{A}$ spreads out (diverges) from the point in question.



if outflow > inflow
Divergence +ve

Outflow = inflow
divergence zero.





inflow > outflow
divergence -ve

Examples :

(i) Gauss law in electrostatics, $\nabla \cdot E = \frac{\rho}{\epsilon_0}$

divergence of electric flux through the gaussian surface is equal to $1/\epsilon_0$ times the volume charge density ρ.

(ii) In magnetism, $\nabla \cdot B = 0$. (Magnetic flux entering is equal to the flux leaving.)

(iii) $\nabla \cdot J = -\frac{\partial \rho}{\partial t}$ (eqn. of continuity)

$\nabla \cdot J$ gives the net outflow of charge through the closed surface that encloses unit volume in unit time. $-\frac{\partial \rho}{\partial t}$ gives the rate of decrease of total charge contained in unit volume in unit time.

Curl and its physical significance

The cross product of differential operator 'del' with a vector point function is known as 'curl' of the function.

$$\nabla \times \vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times \left(\hat{i} A_x + \hat{j} A_y + \hat{k} A_z\right)$$

$$\nabla \times \vec{A} = \hat{i}\left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right] + \hat{j}\left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right] + \hat{k}\left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - \hat{j}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$\nabla \times \vec{A}$ is vector.

$\nabla \times \vec{A}$ is a measure of how much the vector $\vec{A}$ curls around the point in question.

curl $\vec{A}$ is associated with some sort of rotation or circulation.

If $\nabla \times \vec{A} \neq 0$ the vector field must have circulation.

$\nabla \times \vec{A} = 0$, then vector field is said to be irrotational.

eg. $\nabla \times \vec{E} = 0$ in electrostatics

Line Integral

A line integral is an expression of the form,

$\int_a^b \vec{F} \cdot d\vec{l}$ where $\vec{F}$ is a vector function and

$d\vec{l}$ is the infinitesimal displacement vector.

Here integration is to be carried out along a prescribed path C from a to point b.

eg: Workdone $W = \int_a^b \vec{F} \cdot d\vec{l}$, where $\vec{F}$ = Force

$d\vec{l}$ = infinitesimal displacement.

→ For closed path $\oint \vec{F} \cdot dl$

Surface Integral

A surface integral is an expression of the form,

$\int \vec{F} \cdot d\vec{s}$ where $\vec{F}$ is a vector function and

$d\vec{s}$ is an infinitesimal area with direction ⊥cular to the surface S.

For closed surface, $\oint \vec{F} \cdot d\vec{s}$

eg: Total amount of magnetic flux passing through the surface is $\oint \vec{B} \cdot d\vec{s}$

Volume Integral

$\oint \vec{F} \cdot dV$ ⇒ expression for volume integral.

where $\vec{F}$ vector function and dV is infinitesimal volume, ie $dV = dx\,dy\,dz$.

→ Gauss divergence Theorem

It relates surface integral and volume integral.

$$\oint_S \vec{F} \cdot d\vec{s} = \int_V (\nabla \cdot \vec{F})\, dV \quad \Big| \; dv = dx\,dy\,dz.$$

$$\oiint_S \vec{F} \cdot d\vec{s} = \iiint_V (\nabla \cdot \vec{F})\, dV$$

ie volume integral of $\nabla \cdot \vec{F}$ is equal to the surface integral of $\vec{F} \cdot d\vec{s}$ over closed surface.

Stoke's Theorem

Stoke's theorem relates line integral and surface integral. According to stoke's theorem, the surface integral of curl $\vec{F}$ over a surface is equal to the line integral of $\vec{F} \cdot d\vec{r}$ along a closed path around the surface.

$$\boxed{\int_s (\nabla \times \vec{F}) \cdot d\vec{s} = \oint_l \vec{F} \cdot d\vec{r}}$$

or,

$$\iint (\nabla \times \vec{F}) \cdot d\vec{s} = \oint \vec{F} \cdot d\vec{r}$$

⇒ Displacement current and Conduction current

Consider a dielectric placed in an electric field. The bound charges appear at the surface of the dielectric due to Polarization. So dielectric contain free charges and bound charges. If $\vec{E}$ is the field due to all charges (free + bound). Inorder to consider the field due to free charge and bound charge separately we use vectors $\vec{D}$ (due to free charge) and $\vec{P}$ (due to bound charges)

$\vec{D}$ = displacement vector
$\vec{P}$ = Polarization vector

Then $\boxed{\vec{D} = \varepsilon_0 \vec{E} + \vec{P}}$

For free space $\vec{P} = 0$, $\therefore \vec{D} = \varepsilon_0 \vec{E}$

Proof: we know $\nabla \cdot \vec{E} = \rho/\varepsilon_0 = \frac{(\rho_f + \rho_b)}{\varepsilon_0}$

$$\varepsilon_0(\nabla \cdot \vec{E}) = \rho_f + \rho_b \qquad \because \rho_b = -\nabla \cdot \vec{P}$$

$$(\nabla \cdot \varepsilon_0 \vec{E}) = \rho_f - \nabla \cdot \vec{P}$$

$$\nabla \cdot (\underbrace{\varepsilon_0 \vec{E} + \vec{P}}_{\vec{D}}) = \rho_f \Rightarrow \nabla \cdot \vec{D} = \rho_f$$

Displacement current

According to Maxwell, a time-varying electric field in a region produces a magnetic field just like a conduction current in a conductor. We know that a current produces a magnetic field. ie, the time-varying electric field is called displacement current.

Consider the case of charged capacitor, let q be the charge, A the plate area and d, the distance b/w the plates.

Electric field b/w the plates $E = \frac{q}{\varepsilon_0 A}$

$$\varepsilon_0 E = \frac{q}{A} = D \quad \text{or} \quad q = AD$$

$\therefore$ displacement current $\frac{dq}{dt} = I_d = A\frac{\partial D}{\partial t}$

For free space $D = \varepsilon_0 E$

The displacement current density $J_d = \frac{I_d}{A} = \frac{\partial D}{\partial t}$

($J_d \Rightarrow$ current through unit Area)

Conduction current : Conduction current is due to drift of electric charges in a conductor when an electric field is applied.

$I_c = \frac{dQ}{dt}$, $J_c = \frac{I_c}{A}$

Ohm's law gives $V = I_c R$

we know $E = \frac{V}{l} \Rightarrow El = V \Rightarrow El = I_c R$

$El = I_c R \Rightarrow$ | $\because R = \frac{\rho l}{A}$

$E\cancel{l} = I_c \frac{\rho \cancel{l}}{A}$ | $\because \frac{I_c}{A} = J_c$

$E = J_c \rho$

$\boxed{J_c = \frac{E}{\rho} = \sigma E}$ | $\sigma = 1/\rho$

conductivity σ,
ρ = resistivity

Conduction current	Displacement current
It obey ohm's law	It does not obey ohm's law.
It is due to flow of electric charges	It is due to electric field that changes with time.
$J_c = \frac{I_c}{A} = \sigma E$	$J_d = \frac{I_d}{A} = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}(\epsilon_0 E)$ $I_d = \epsilon \omega E A$ - depends on frequency of the wave.

$\Rightarrow$ Equation of continuity

charge in a volume V is,

$Q(t) = \int_V \rho \, d\tau \longrightarrow (1)$ $\Rightarrow \rho \Rightarrow$ volume charge density, $\rho(r,t)$

$Q(t) = \int_V \rho(r,t) \, d\tau \longrightarrow (2)$ $d\tau = dx\,dy\,dz$

current flowing out through the boundary S is, $\int_S J \cdot da$

$\frac{I}{A} = J$
$I = J \cdot A$
$dI = J \cdot da$

$$\text{current} = \int_S J \cdot da$$

$$\frac{dQ}{dt} = -\int_S J \cdot da \longrightarrow (3)$$

From (2) $\Rightarrow$ $$\frac{dQ}{dt} = \int \frac{\partial \rho}{\partial t} \, d\tau \longrightarrow (4)$$

$\because Q(t) = \int \rho \, d\tau$

$\frac{dQ}{dt} = \frac{d}{dt} \int \rho \, d\tau$

$= \int \frac{\partial \rho}{\partial t} \, d\tau$

$\rho(r, t)$

~~comparing (3) & (4)~~

using Gauss divergence theorem on (3)

$$-\int_S J \cdot da = -\int (\nabla \cdot J) \, d\tau \longrightarrow (5)$$

$d\tau \to$ volume element
$d\tau = dx\,dy\,dz$.

$\therefore$ From (4) & (5)

$$\frac{\partial \rho}{\partial t} = -(\nabla \cdot J) \Rightarrow \boxed{(\nabla \cdot J) = -\frac{\partial \rho}{\partial t}}$$

imp $\Rightarrow$

Maxwell's Equations

The electromagnetic wave phenomena are governed by a set of four equations known as Maxwell's field equations. Maxwell restated Gauss laws in electricity and magnetism and Faraday's law of electromagnetic induction.

He modified Ampere's circuital law by introducing the concept of displacement current.

Maxwell's equations are,

(i) $\nabla \cdot \vec{D} = \rho$

(ii) $\nabla \cdot \vec{B} = 0$

(iii) $\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$

(iv) $\nabla \times H = J + \frac{\partial \vec{D}}{\partial t}$

I $\nabla \cdot \vec{D} = \rho$ - Gauss law in electrostatics

Gauss law in electrostatics: The electric flux from a closed surface is equal to $1/\epsilon_0$ times the charge enclosed by the surface.

$$\oint \vec{E} \cdot d\vec{s} = q/\epsilon_0$$

$$\oint \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_0} \int \rho \, dv \longrightarrow (1)$$

$\rho \Rightarrow$ volume charge density.

$$\epsilon_0 \oint \vec{E} \cdot d\vec{s} = \int \rho \, dv \longrightarrow (2)$$

$$\oint \epsilon_0 \vec{E} \cdot d\vec{s} = \int \rho \, dv \longrightarrow (3)$$

$$\oint \vec{D} \cdot d\vec{s} = \int \rho \, dv \longrightarrow (4)$$

where $\vec{D} = \epsilon_0 \vec{E}$, In free space

Apply Gauss divergence theorem on LHS

$$\oint \vec{D} \cdot d\vec{s} = \int_v (\nabla \cdot D) \, dV \longrightarrow (5)$$

combine (4) & (5),

$$\int_V (\nabla \cdot \vec{D})\, dV = \int_V \rho\, dV$$

ie, $\boxed{\nabla \cdot \vec{D} = \rho}$

If the medium is dielectric,

$$\oint \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_0}\left[\int \rho\, dv + \int \rho_b\, dv\right]$$

$$\oint \epsilon_0 \vec{E} \cdot d\vec{s} = \left[\int \rho\, dv - \int (\nabla \cdot \vec{P})\, dv\right]$$

$$\oint \epsilon_0 \vec{E} \cdot d\vec{s} + \int (\nabla \cdot P)\, dv = \int \rho\, dv$$

$$\int (\nabla \cdot \epsilon_0 E)\, dv + \int (\nabla \cdot P)\, dv = \int \rho\, dv$$

$$\int \nabla \cdot (\underbrace{\epsilon_0 \vec{E} + \vec{P}}_{D})\, dv = \int \rho\, dv$$

$$\int (\nabla \cdot D)\, dv = \int \rho\, dv$$

II $\boxed{\nabla \cdot B = 0}$ Gauss law in magnetism.

Gauss law in magnetism states that the total magnetic flux through a closed surface is zero. This follows from the fact that the lines of $\vec{B}$ are always ~~zero~~ closed.

$$\phi_B = \oint B \cdot ds = 0$$

This implies that a monopole or an isolated magnetic pole cannot exist to serve as a source or sink for the line of magnetic induction $\vec{B}$.

using Gauss divergence theorem,

$$\int (\nabla \cdot B)\, dV = 0$$

ie, $\nabla \cdot B = 0$

III $\boxed{\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}}$ — Faraday's law of electro-magnetic induction.

From Faraday's law,

$$e = -\frac{d\phi_B}{dt} \longrightarrow (1)$$

Magnetic flux linked with an area $d\vec{s}$ is,

$$\phi_B = \int_s \vec{B}\cdot d\vec{s} \longrightarrow (2)$$

$$\therefore e = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$$

Induced emf is related to the corresponding electric field as,

electrostatics

$$\frac{V}{l} = E$$

$$V = E \cdot l.$$

$$e = \int_c \vec{E}\cdot d\vec{l}$$

$$\int \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$$

$$\int \vec{E}\cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$$

Apply stoke's theorem on LHS.

$$\int \vec{E}\cdot d\vec{l} = \int (\nabla \times \vec{E})\cdot d\vec{s}$$

$$\int (\nabla \times \vec{E})\cdot d\vec{s} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$$

$$\therefore \boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}.$$

IV $\nabla \times B = J + \frac{\partial \vec{D}}{\partial t}$ (Modified Ampere's law)

Ampere's law is $\oint \vec{B}\cdot d\vec{l} = \mu_0 I \longrightarrow (1)$

we know $\vec{B} = \mu_0 \vec{H}$

$(1) \Rightarrow \quad \mu_0 \oint H \cdot dl = \mu_0 I \longrightarrow (2)$

$$\oint H \cdot dl = I \longrightarrow (3)$$

We know $I = \int_S \vec{J} \cdot d\vec{s}$ | $\frac{I}{A} = J$

$$(3) \Rightarrow \oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{s} \longrightarrow (4)$$

Apply stoke's theorem on LHS,

$$\oint \vec{H} \cdot d\vec{l} = \int (\nabla \times \vec{H}) \cdot d\vec{s}$$

$$(4) \Rightarrow \int (\nabla \times H) \cdot d\vec{s} = \int \vec{J} \cdot d\vec{s} \longrightarrow (5)$$

$$\therefore \nabla \times H = J$$

Inconsistency of Amper's law:

Maxwell showed that Ampere's law is inconsistent.

$$\nabla \times H = J$$

Taking divergence on both sides,

$$\nabla \cdot (\nabla \times H) = \nabla \cdot J \longrightarrow (6)$$

LHS of (6) $\Rightarrow 0$ ie, $\nabla \cdot (\nabla \times H) = 0$.

$\therefore (6) \Rightarrow \nabla \cdot J = 0$. $\Rightarrow$ It violates eqn. of continuity

ie from equation of continuity,

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$$

when $\nabla \cdot J = 0 \Rightarrow -\frac{\partial \rho}{\partial t} = 0$, $\rho =$ const.

$\therefore$ Ampere's circuital law is true for steady currents and wrong for time varying currents.

Maxwell's Modification

James clerk Maxwell removed the inconsistency of Ampere's law by introducing the concept of displacement current.

Ampere's circuital law tells us that magnetic field is caused by conduction current only. Maxwell discovered that there is one more source of magnetic field, which is the time varying electric field. The time varying electric field is called displacement current.

Displacement current density is J_d

$\therefore \nabla \times H = J_c + J_d \longrightarrow (1)$ $\quad J_c$ = conduction current density

$\nabla \cdot (\nabla \times H) = \nabla \cdot (J_c + J_d) \longrightarrow (2)$ $\quad J_d$ = displacement current density

$\boxed{\nabla \cdot (\nabla \times H) = 0}$

$\oint \vec{E} \cdot d\vec{s} = q/\varepsilon_0$

$\oint \varepsilon_0 E \cdot d\vec{s} = q$

$\oint D \cdot ds = q$

$\oint D \cdot ds = \int \rho \, dv$

$\oint (\nabla \cdot D) dv = \int \rho \, dv$

$\boxed{\nabla \cdot D = \rho}$

$\therefore \nabla \cdot (J_c + J_d) = 0$

$\nabla \cdot J_c = -\nabla \cdot J_d$

or,

$\nabla \cdot J_d = -\nabla \cdot J_c$

$\nabla \cdot J_d = \dfrac{\partial \rho}{\partial t}$

$\nabla \cdot \vec{J_d} = \dfrac{\partial (\nabla \cdot \vec{D})}{\partial t}$

$\nabla \cdot \vec{J_d} = \nabla \cdot \dfrac{\partial \vec{D}}{\partial t}$

$\therefore \vec{J_d} = \dfrac{\partial \vec{D}}{\partial t}$

$\therefore \boxed{\nabla \times H = J_c + \dfrac{\partial \vec{D}}{\partial t}}$

From eqn. of continuity
$\nabla \cdot J = -\partial \rho / \partial t$
where J is J_c

$\because \rho = \nabla \cdot \vec{D}$

where $\dfrac{\partial \vec{D}}{\partial t} = \dfrac{\partial (\varepsilon_0 \varepsilon_r E)}{\partial t}$

$\dfrac{\partial \vec{D}}{\partial t} = \varepsilon_0 \varepsilon_r \dfrac{\partial \vec{E}}{\partial t}$

$\boxed{\dfrac{\varepsilon}{\varepsilon_0} = \varepsilon_r}$

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Maxwell's equation for free space.

We know the Maxwell's eqns are,

(i) $\nabla \cdot D = \rho$

(ii) $\nabla \cdot B = 0$

(iii) $\nabla \times E = -\dfrac{\partial B}{\partial t}$

(iv) $\nabla \times H = J_c + \dfrac{\partial D}{\partial t}$

For free space, conductivity $\sigma = 0$ and charge density $\rho = 0$. Hence constitutive relations become,

$\vec{J_c} = \sigma \vec{E} = 0.$

$\vec{D} = \epsilon_0 E + P = \epsilon_0 E$

$\vec{B} = \mu_0 \vec{H}$

ϵ_0 = permittivity
μ_0 = permeability

$\therefore$ (i) becomes, $\nabla \cdot \epsilon_0 E = \rho = 0$

$\epsilon_0 (\nabla \cdot E) = 0$

$\therefore \boxed{(\nabla \cdot E) = 0} \longrightarrow (1)$

(ii) becomes, $\nabla \cdot \mu_0 H = 0$

$\mu_0 (\nabla \cdot H) = 0 \Rightarrow \boxed{\nabla \cdot \vec{H} = 0} \longrightarrow (2)$

(iii) becomes, $\nabla \times E = -\dfrac{\partial}{\partial t} \mu_0 H$

$\boxed{\nabla \times E = -\mu_0 \dfrac{\partial H}{\partial t}} \longrightarrow (3)$

(iv) becomes, $\nabla \times H = 0 + \dfrac{\partial}{\partial t} \epsilon_0 E$ $\quad \because J_c = \sigma E = 0$

$\boxed{\nabla \times H = \epsilon_0 \dfrac{\partial \vec{E}}{\partial t}} \longrightarrow (4)$

$\because$ For Free space, $\nabla \cdot E = 0$
$\nabla \cdot H = 0$

$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$

$\nabla \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

Velocity of electromagnetic waves in free space:

Consider (3) of Maxwell's eqn for freespace,

$$\nabla \times E = -\mu_0 \frac{\partial H}{\partial t} \longrightarrow (5)$$

Taking curl on both sides,

We have,
$$\boxed{\nabla \times \nabla \times E = \nabla(\nabla \cdot E) - \nabla^2 E}$$

$$\nabla \times \nabla \times E = \nabla \times \left(-\mu_0 \frac{\partial H}{\partial t}\right)$$

$$\nabla(\nabla \cdot E) - \nabla^2 E = \nabla \times \left(-\mu_0 \frac{\partial H}{\partial t}\right) \longrightarrow (6)$$

$\because$ $\nabla \cdot E = 0$ For free space

$$(6) \Rightarrow \quad -\nabla^2 E = -\mu_0 \left(\nabla \times \frac{\partial H}{\partial t}\right) \longrightarrow (7)$$

$$-\nabla^2 E = -\mu_0 \frac{\partial}{\partial t}(\nabla \times H) \longrightarrow (8)$$

$$\because \nabla \times H = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$(8) \Rightarrow -\nabla^2 E = -\mu_0 \frac{\partial}{\partial t}\left(\varepsilon_0 \frac{\partial E}{\partial t}\right) = -\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\therefore \nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \longrightarrow (9)$$

Similarly taking curl on, $\nabla \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ and do the same procedure we get,

$$\nabla^2 H = \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2} \longrightarrow (10)$$

Eqn (9) and (10) tell us that E and H propagates as waves.

(9) and (10) are the wave equations for $\vec{E}$ and $\vec{H}$. Comparing these equations with classical wave eqn,

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \qquad | v = \text{Velocity of wave}$$

we get $\frac{1}{v^2} = \mu_0 \varepsilon_0 \Rightarrow v^2 = \frac{1}{\mu_0 \varepsilon_0}$

$$\therefore v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Maxwell obtained velocity of light as 3.107×10^8 m/s.

Poynting's Theorem

Electromagnetic waves carry energy and momentum. The energy transferred from source to distant point is computed by using Poynting's theorem. Poynting's theorem state that the vector

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B})$$

at any point is a measure of the rate of energy flow per unit area at that point. The vector $\vec{S}$ is called poynting vector.

Let $\vec{E} = \hat{i} E_0 \sin(kz - \omega t)$

$\cancel{\vec{B} = \hat{j} B_0}$ $\vec{B} = \hat{j} B_0 \sin(kz - \omega t)$

$$\vec{S} = \vec{E} \times \vec{H} = \frac{1}{\mu_0} (\vec{E} \times \vec{B})$$

$$= \frac{1}{\mu_0} E_0 \sin(kz - \omega t)\, B_0 \sin(kz - \omega t)(\hat{i} \times \hat{j})$$

$$\vec{S} = \frac{E_0 B_0}{\mu_0} \sin^2(kz - \omega t)\, \hat{k}$$

$\vec{S}$ along z direction.

$\langle \vec{S} \rangle$ = average value of poynting's vector

$$\langle \vec{S} \rangle = \frac{E_0 B_0}{\mu_0} \langle \sin^2 (kz - \omega t) \rangle$$

Average value of $\sin^2 kz - \omega t$ is $1/2$

$$\therefore \langle \vec{S} \rangle = \frac{1}{2} \frac{E_0 B_0}{\mu_0}.$$

We have $\frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \eta \Rightarrow$ characteristic impedance (ohm).

We have $v^2 = \frac{1}{\mu_0 \varepsilon_0} \Rightarrow \varepsilon_0 = \frac{1}{v^2 \mu_0} = \frac{1}{c^2 \mu_0}.$

$$\frac{E^2}{H^2} = \frac{\mu_0}{\varepsilon_0} = \frac{\mu_0}{\frac{1}{c^2 \mu_0}} = \mu_0^2 c^2$$

$$\frac{E^2}{H^2} = \mu_0^2 c^2 \Rightarrow E^2 = \mu_0^2 H^2 c^2 = B^2 c^2.$$

$$\boxed{E = BC}$$